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MODELING OF MHD BLOOD FLOW IN A BALLOON CATHETERIZED ARTERIAL STENOSIS WITH THERMAL RADIATION

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ABSTRACT
This article is concerned with the analysis of blood flow through arterial catheterization (widening of arteries by a balloon in the presence of mild stenosis), when blood is represented by MHD Newtonian fluid. The mild stenosis approximation is used to solve the problem. A uniform magnetic field is applied perpendicular to the porous artery. A mathematical model is developed and analyzed by using appropriate mathematical techniques. The dimensionless governing coupled, non-linear partial differential equations are solved numerically by finite difference method. This study provides a scope for estimating the influence of the problem parameters on different flow characteristics and to determine which of the parameters has the most dominating role. The study of the above model is very significant as it has direct applications in the treatment of cardiovascular diseases.

KEYWORDS: Blood flow; Catheterization; Stenosis; Radiation.

I. INTRODUCTION
The theoretical and experimental investigations on flow of blood are vital for the diagnosis of cardiovascular diseases and development of pathological designs in human or animal physiology. The gradual thickening of an artery has long been understood as an early process in the formation of atherosclerosis and one of the most wide-spread diseases in humans leading to the malfunction of the cardiovascular system. Atherosclerosis is a major underlying cause of angina and myocardial infarction. In atherosclerotic arteries, the opening is typically narrowed and the wall is stiffened by the build-up of plaque with a lipid core and a fibromuscular cap. It is established that blood flow in an arterial stenosis disturbs the flow pattern in circulatory system and stenosis or arteriosclerosis is one of the commonly arising cardiovascular diseases (Young [1]). Although, the reason for the initiation of such diseases is yet to be understood in totality, there is a concrete faith that hemodynamic features and mechanical behavior of blood vessels walls play a pivotal role in the instigation formation, and proliferation of the disease (Caro [2]; Young and Tsai[3]). Various researchers have stated mathematical treatment in the flow of blood in arteries subject to numerous physiologic conditions [4-6]. The recent useful contributions to that subject are referenced in the literature [7-13].

In recent times, with the evolution of coronary balloon angioplasty, there has been a considerable increase in the use of catheters of various sizes. Catheters are medical devices that can be inserted in the body to treat diseases or perform a surgical procedure. By modifying the material or adjusting the way catheters are manufactured. Also, it is possible to tailor catheters for cardiovascular, urological, gastrointestinal, neurovascular, and ophthalmic applications. Catheters can be inserted into a body cavity, duct, or vessel. The process of inserting a catheter is catheterization. Insertion of a catheter into the lumen of the arterial stenosis upshots substantial deviations in the blood flow pattern. In view of this, Mekheimer and Kot [14]) considered the effects of tapering in arterial wall and catheter size on blood flow respectively. During the process of angioplasty, the translational pressure gradient was measured by Ganz et al.[15]. Sarkar and Jayaraman [16] have observed the troubled flow pattern in the stenosed artery due to catheterization. In balloon catheter technique, a catheter with an inflatable balloon at its tip which is used during a catheterization procedure to enlarge a narrow opening or passage within the artery. The catheter is carefully guided to the location at which stenosis occurs in coronary arteries and then balloon is then inflated to fracture the fatty deposits and widen the narrowed portion.
During the last few decades, extensive research work has been done on the dynamics of biological fluid in the presence of magnetic field with implications in the bioengineering and medical technology. The development of

magnetic devices for cell separation, targeted transport of magnetic particles as drug carriers, magnetic wound or cancer tumour treatment causing magnetic hyperthermia, reduction of bleeding during surgeries or provocation of occlusion of the feeding vessels of cancer tumours and the development of magnetic tracers, as well are well-known applications in this domain of research. The most characteristic biological fluid is blood, which behaves as a magnetic fluid, due to the complex interaction of the intercellular protein, cell membrane and haemoglobin as a form of iron oxides which is present at a uniquely high concentration in the mature red blood cells, while its magnetic properties are affected by factors such as the state of oxygenation.

Haik et al. [17] reported a 30% decrease in blood flow rate when subjected to a high magnetic field of 10 T while Yadav et al. [18] showed a similar reduction in blood flow rate but at a much smaller magnetic field of 0.002 T. Sharma et.al [19-20] MHD free convective flow of a viscous fluid past an infinite vertical porous. The radiation effect in blood is of significant interest to clinicians in the therapeutic procedure of hyperthermia, which has a well-recognized effect in oncology. Its effect is achieved by overheating the cancerous tissues Szasz [21]) by means of electromagnetic radiation. Different aspects of hyperthermia were discussed/studied by several researchers (Abe and Hiraoka [22], Molls[23], Sharma et al [24-26]). Hence, the purpose of study to analyse the arterial catheterization (widening of arteries by a balloon in the presence of mild stenosis), in the presence of thermal radiation. The effects of various parameter is discussed numerically and explained graphically.

II. MATERIALS AND METHODS

Let (r, θ, x) be the cylindrical polar coordinate system with $r = 0$ as the axis of symmetry of the tube. Consider the flow of an incompressible magnetohydrodynamic (MHD) Newtonian fluid of constant viscosity μ and density ρ through coaxial tubes such that the outer tube with an axially non-symmetric but radially symmetric mild stenosis having length L and the inner tube have a balloon (angioplasty) on its wall and assume that the balloon model is axi-symmetric in nature and u and w are the velocity component in r and z directions, respectively. The stenosed wall and the balloon model are defined by the functions $h(z)$ and $R(z)$ respectively.

The geometry of the stenosis is defined as Mekheimer and El Kot [23]

$$h(z) = d(z)[1 - \eta(b^{n-1}(z-a) - (z-a)^n)], \; a \le z \le a+b$$
$$= d(z), \text{ otherwise} \qquad (1)$$

with $d(x) = d_0 + \varepsilon z$, where $d(z)$ is the radius of the tapered arterial segment in the stenotic region, d_0 is the radius of the non-tapered artery in the non-stenotic region, is the tapering parameter, b is the length of the stenosis, $n(\ge 2)$ is a parameter determining the shape of the constriction profile and referred to as the shape parameter (the symmetric stenosis occurs for $n = 2$) and a indicates its location.

The geometry of the catheter is defined as Mekheimer and El Kot [14]

$$R(z) = d_0[k + \delta_2 \exp(-\pi^2(z - z_d - 0.5)^2], \; a \le z \le a+b$$
$$= kd_0, \quad \text{otherwise} \qquad (2)$$

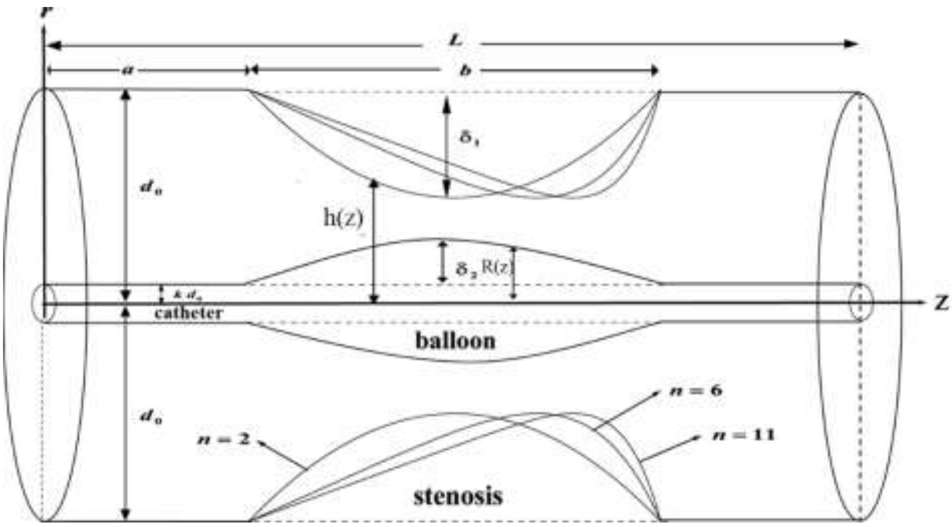

Figure 1(a): The geometry of stenosis in inserted catheterized artery having a balloon

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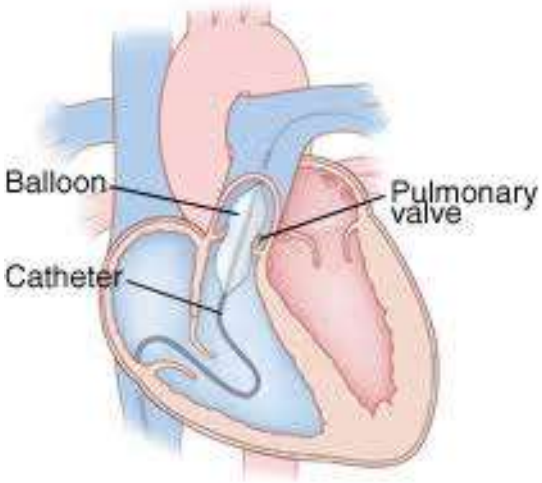

Figure 1(b): Catheterized artery having a balloon

The fluid is assumed to be slightly conducting, so that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison with the applied magnetic field. It is also assumed that there is no applied voltage, so that electric field is absent. All the fluid properties are assumed to be constant except that of the influence of the density variation with temperature and concentration in the body force term. A first-order homogeneous chemical reaction is assumed to take place in the flow.
The equations governing the steady incompressible Newtonian fluid with energy and mass concentration equation are given as

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (3)$$

$$\rho\left[\bar{u}\frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{r}}\right] = -\frac{\partial \bar{p}}{\partial \bar{r}} + \frac{\partial}{\partial r}\left[2\mu\frac{\partial \bar{u}}{\partial \bar{r}}\right] + \frac{2\mu}{\bar{r}}\left(\frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\bar{u}}{\bar{r}}\right) + \frac{\partial}{\partial \bar{z}}\left[\mu\left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}}\right)\right] \quad (4)$$

$$\rho\left[\bar{u}\frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}}\right] = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{\partial}{\partial \bar{z}}\left[2\mu\frac{\partial \bar{w}}{\partial \bar{z}}\right] + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left[\mu\bar{r}\left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}}\right)\right] - \sigma_1\mu_0^2 H_0^2 \bar{w} \quad (5)$$

$$\rho c_p\left[\bar{u}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}\right] = \frac{k}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{T}}{\partial \bar{r}}\right) - \frac{\partial q_r}{\partial \bar{z}} \quad (6)$$

Boundary Conditions

$\bar{w} = \bar{T} = 0 \;\; at\, r = R(z)$

$\bar{w} = \bar{T} = 0 \; at\, r = h(z)$, (7)

where $R(z) = d_0[k + \delta_2 \exp(-\pi^2(z - z_d - 0.5)^2], a \le z \le a + b$

and $h(z) = d(z)[1 - \eta(b^{n-1}(z-a) - (z-a)^n)]$.

Equations are converted into dimensionless form by introducing non dimensional parameters. Introducing following dimensionless parameters:

$$r = \frac{r}{d_0}, z = \frac{\bar{z}}{b}, w = \frac{\bar{w}}{u_0}, u = \frac{b\bar{u}}{u_0\delta}, Re = \frac{\rho b u_0}{\mu}, M = \sigma_1\mu_m H_0\sqrt{\frac{\sigma_1}{\mu}},$$

$$R = \frac{k^* k_\infty}{4\sigma^* T^3}\, \sigma = \frac{a}{b}, \delta_1^* = \frac{\delta_1}{d_0} \quad (8)$$

and by adopting additional conditions (Mekheimer and El Kot [27])

$$Re\frac{\delta n^{\frac{1}{n-1}}}{b} \ll 1$$

$$\frac{d_0 n^{\frac{1}{n-1}}}{b} \sim O(1) .$$

The dimensionless equations take the following form for the case of mild stenosis ($\frac{\delta}{d_0} \ll 1$)

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (9)$$

$$\frac{\partial p}{\partial r} = 0 ,$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial w}{\partial r}\right)\right] - M^2 w, \quad (10)$$

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$$\left(1+\frac{4}{3R}\right)\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial\theta}{\partial r}\right)\right]=0 \quad , \tag{11}$$

with boundary conditions

$$w=0,\ \theta=0,\quad at\ r=R(z)$$
$$w=0,\ \theta=0,\ \ at\ r=h(z) \tag{12}$$

$$where\ R(z)=\begin{cases}k+\delta_2\exp\left[-\pi^2b^2\left(z-\frac{z_d+0.5}{b}\right)^2\right], \sigma<z<\sigma+1,\end{cases}$$
$$h(z)=(1+\varepsilon z)[1-\eta_1((z-\sigma)-(z-\sigma)^n)]$$

Solution of the Problem:

The above non-linear dimensionless partial differential equations with boundary conditions have been solved numerically by applying explicit finite difference method. The discretization for first order derivatives terms are based on the first order forward difference scheme and for second order terms are based on central difference scheme. To obtain the difference equations, the region of the blood flow is divided into a grid or mesh lines. Solution of these difference schemes are obtained at the intersection of these mesh lines called nodes. Axis is discretized by incrementing *j* and radial direction by incrementing *i*. The finite difference equations at every internal nodal point on a particular n-level constitute a tri diagonal system of equations. These equations are solved using Thomas Algorithm MATLAB.

$$\left(\frac{\partial p}{\partial z}\right)_{i,j}=\frac{1}{r_{i,j}}\frac{w_{i+1,j}-w_{i,j}}{\Delta r}+\frac{w_{i+1,j}-2w_{i,j}+w_{i-1,j}}{(\Delta r)^2}-(M^2)w_{i,j} \tag{13}$$

$$\left(1+\frac{4}{3R}\right)\left(\frac{1}{r_{i,j}}\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta r}+\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta r)^2}\right)=0\ . \tag{14}$$

Results and Discussion:

To observe the quantitative effects of the maximum height attained by the balloon δ_2, the axial displacement of the balloon z_d, the radius of the inner tube, which keeps the balloon in position *k*,, computational procedure is applied (MATLAB) for numerical evaluations of the analytic results. In order to get physical insight, the numerical calculations for the axial velocity, temperature and shear stress for various parameters have been carried out. Values chosen for numerical solutions are $M = 0.2$, $z_d = 0$, $z = 0.9$, $n = 2$, $\delta_2 = 0.2$ and $\delta = 0.3$,$k = 0$ and other dimensionless parameters are varied to understand blood flow behavior.

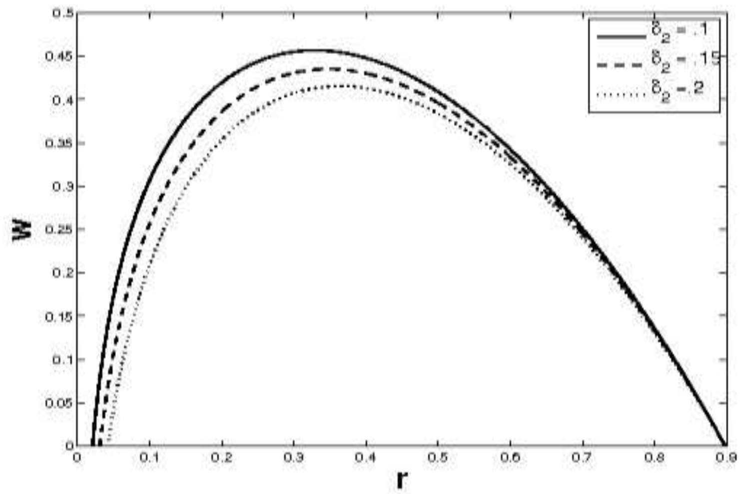

Figure 2: Velocity profile for varying δ_2

It is found that the velocity decreases with increase in the effective size of the catheter as can be seen from the increase in magnitude of the velocity profile with changes in geometrical parameters of the catheter. Velocity is found to decrease with increase in magnetic field, as magnetic field introduces Lorentz force in electrically conducting fluid. This force acts against blood flow which reduces velocity with increase in magnetic field. It is seen that converging artery has higher axial velocity as compared to non tapered and diverging. It is also observed from Figures that the velocity profile increases with decreasing z_d, δ_2, δ, and k .

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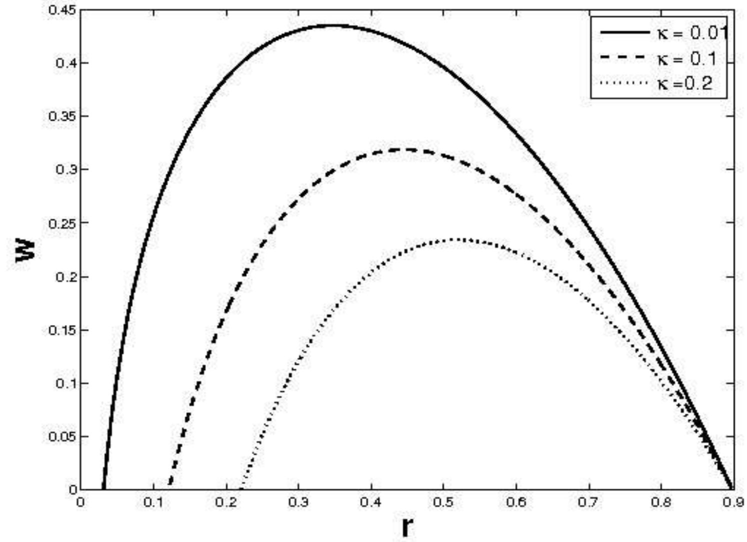

Figure 3: Velocity profile for varying k

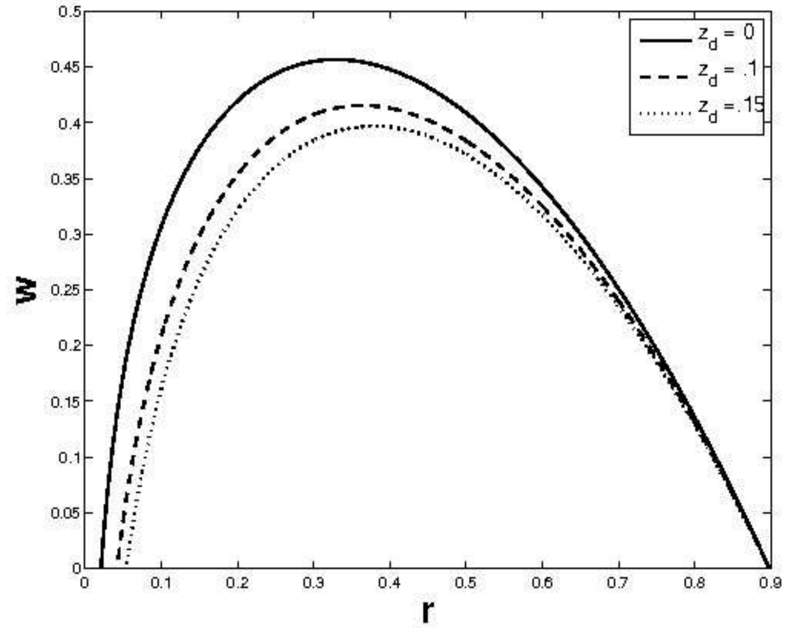

Figure 4: Velocity profile for varying z_d

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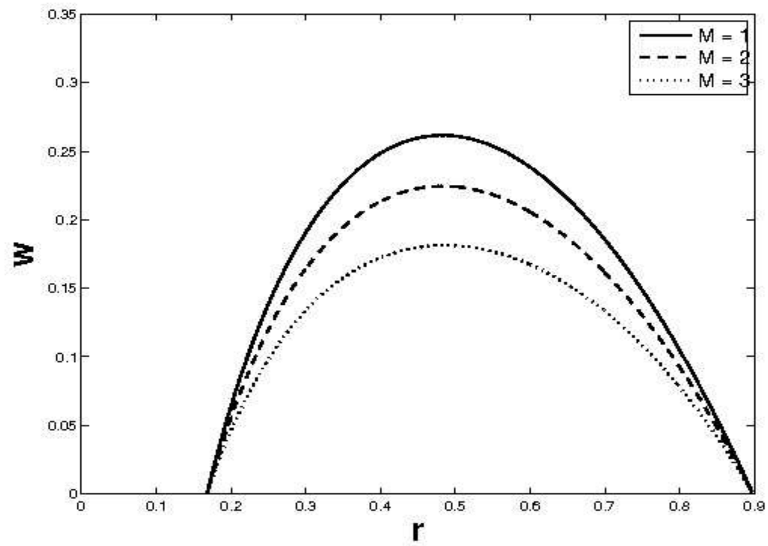

Figure 5: Velocity profile for varying *M*

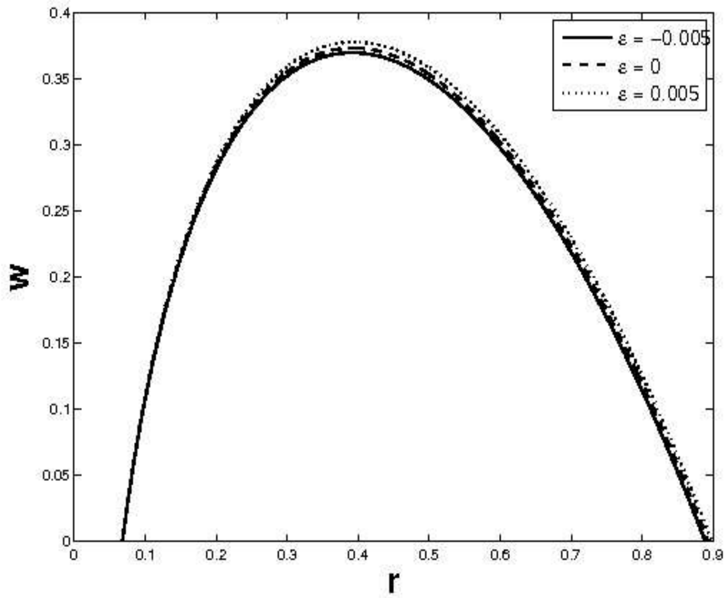

Figure 6: Velocity profile for tapering angle

Shear Stress

The nonzero dimensionless shear stress is given by

$$\tau_s = \left(\frac{\partial w}{\partial r}\right)_{r=R-\delta} \quad (15)$$

Shear stress against δ is plotted for different values of δ_2, z_d and k are drawn by taking default values $M = 0.2$, $z_d = 0, z = 0.9, n = 2, k = 0$

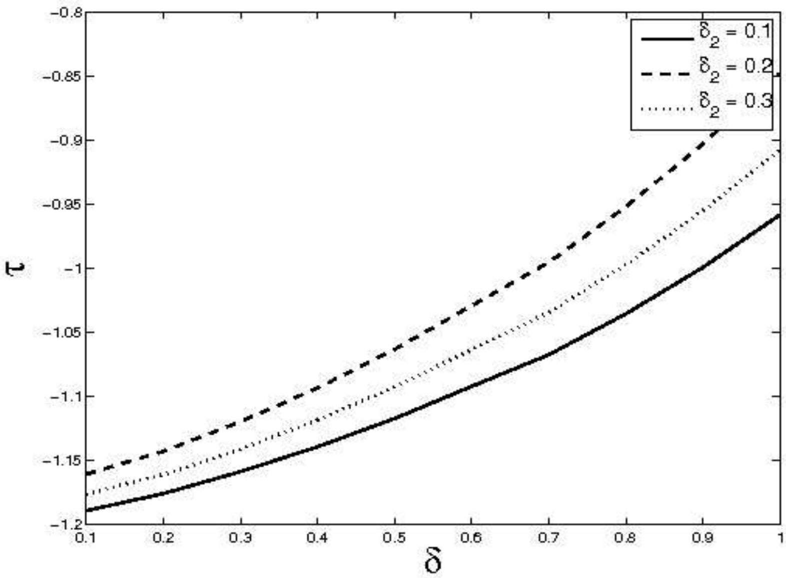

Figure 7: Shear stress for varying δ_2

It is observed that shear stress increases with increase in δ_1 and the magnitude may be dependent on the other geometrical factors.

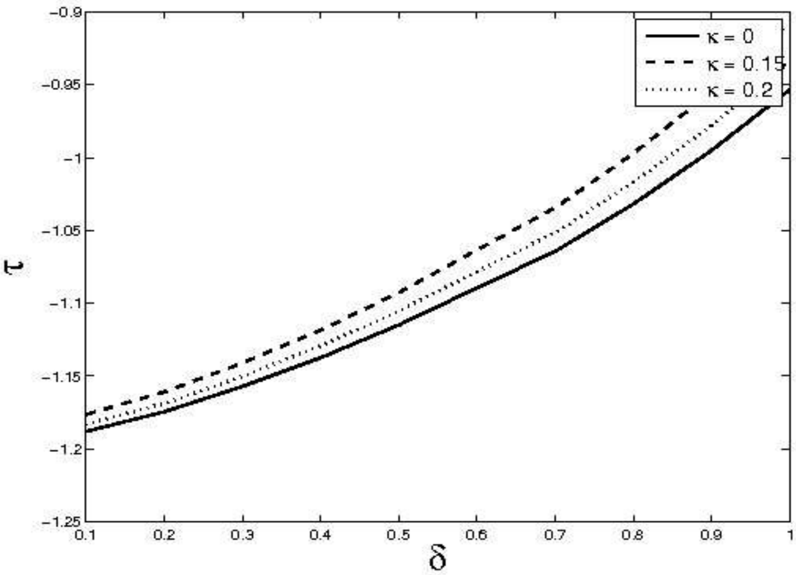

Figure 8: Shear stress for varying *k*

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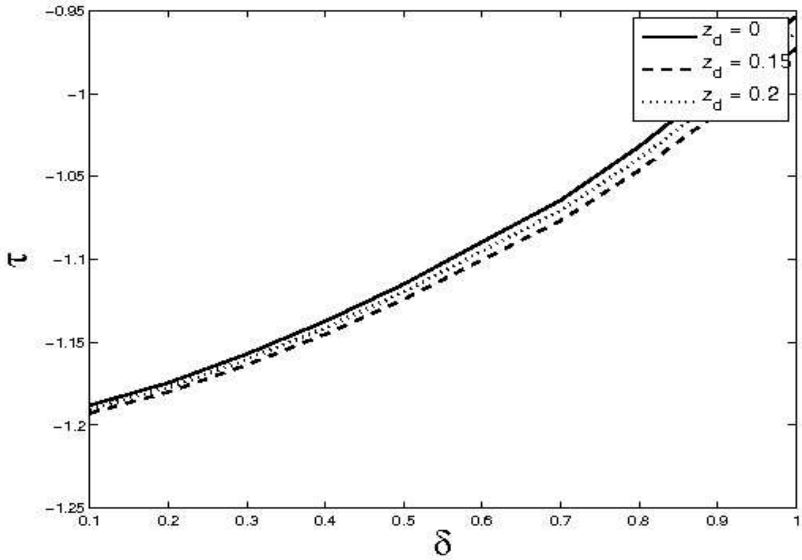

Figure 9: Shear stress for varying z_d

The variation of Temperature profile with different values of δ_2, z_d and k are displayed from Figures (10)-(13). We have taken default values parameters as M = 2, z = 0.9, z_d = 0.1 , k = 0.1.

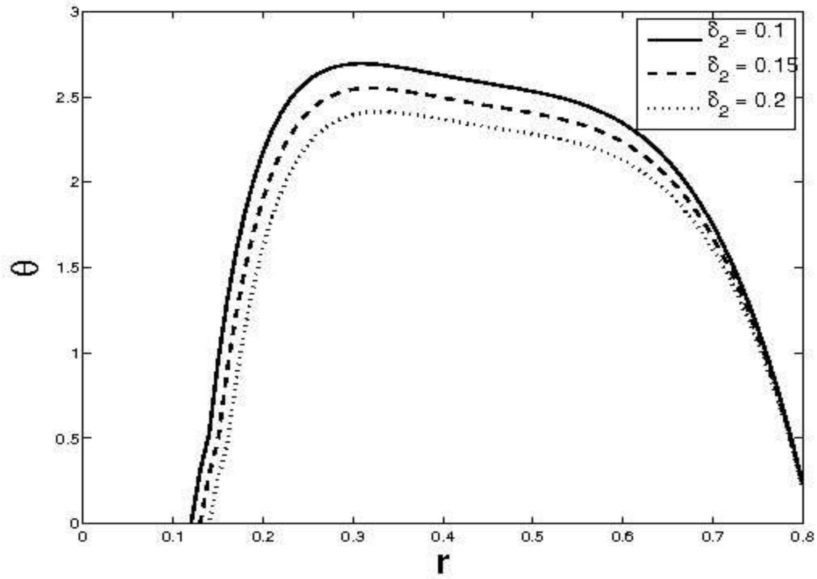

Figure 10: Temperature profile for varying δ_2

It is noted that the temperature profile magnitude drops with increase in δ_2,z_d and k. It is observed from that temperature of blood in presence of stenosis decreases with increase in magnetic field. It is also observed that temperature increases with increase in radiation parameter R as shown in Fig. 13. The radiation acts as a heat source within the blood, the arterial blood temperature (and hence also that along the centreline) should gradually increase with increasing radiation dosage.

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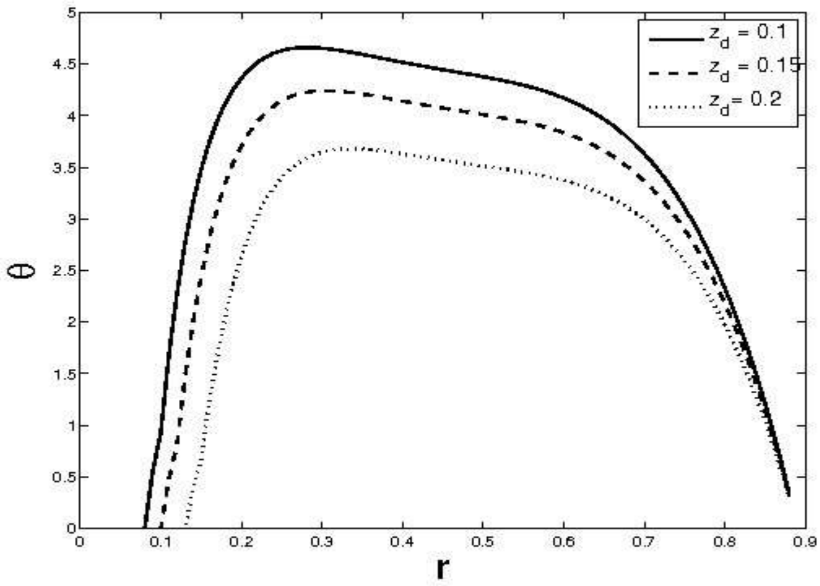

Figure 11: Temperature profile for varying z_d

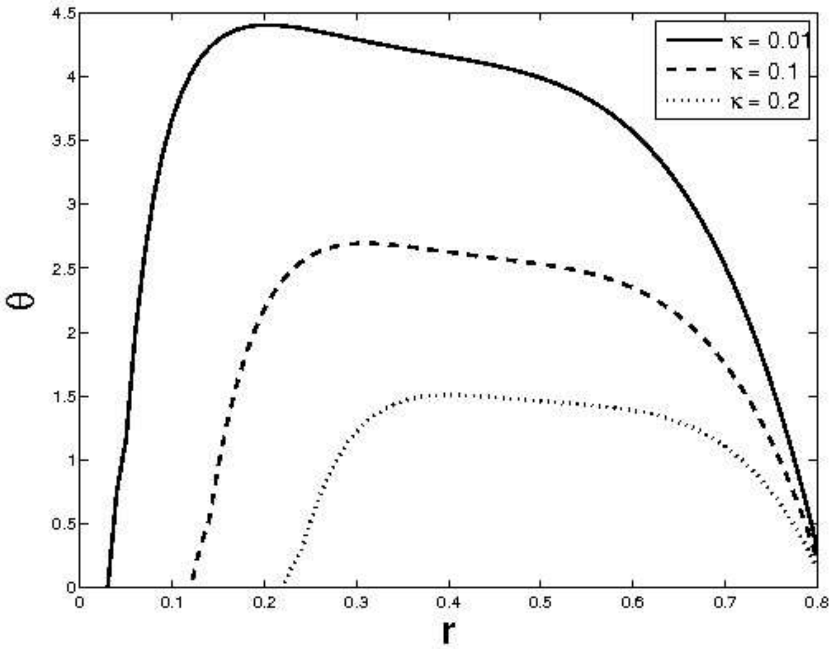

Figure 12: Temperature profile for varying k

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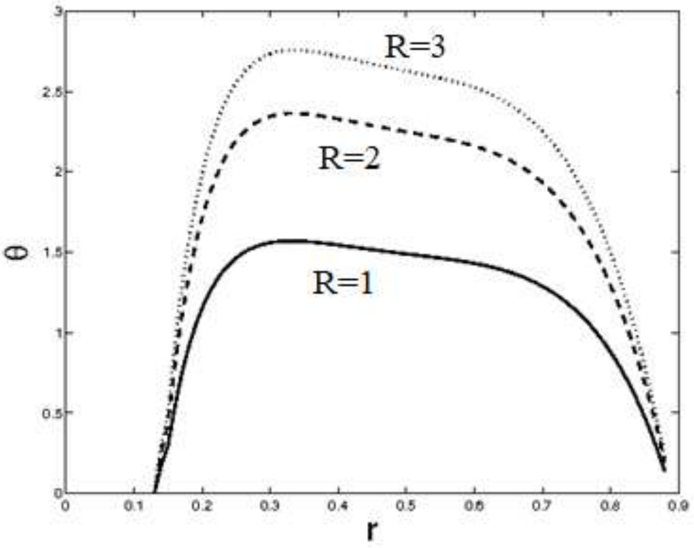

Figure 13: Temperature profile for varying *R*

III. CONCLUSION

An analysis of blood flow through arterial catheterization when blood is represented by MHD Newtonian fluid. The mild stenosis approximation is used to solve the problem. The study of the above model is very significant as it has direct applications in the treatment of cardiovascular diseases. It is observed that temperature of blood in presence of stenosis decreases with increase in magnetic field. It is also noted that temperature increases with increase in radiation parameter.

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